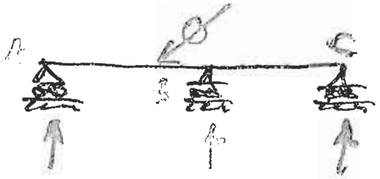
Stability & Instability

A structure is stable, in general, unless the following situations exist:

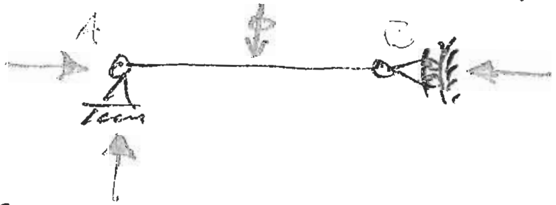
- Reactions are parallel:



This structure will move sidewards under any lateral load. This model is of a 3-axle truck!

- Reactions are concurrent:

If all reactions pass through the same point, the structure will rotate about that point:



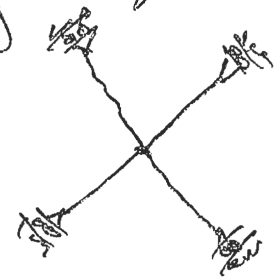
This structure will rotate about point A and cannot take any vertical load as shown.

The structures given above for are termed mechanisms: they will move indefinitely. Another type of mechanism, sometimes called "critical" exists when a structure will not move indefinitely but is not serviceable as a structure:

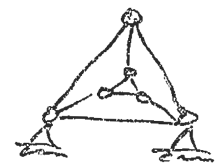
limited amount of large deflection as members go into tension.

Examples:

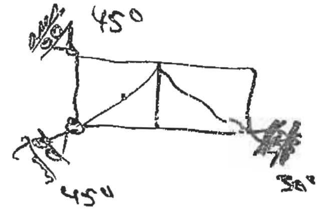
a)



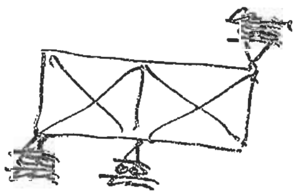
b)



c)



d)



Statical Determinacy & Indeterminacy.

In general, for beams, frames & overall stability of trusses,

$R - C < 3$ – Unstable

$R - C = 3$ – ~~Stable &~~ Stat. Det.

$R - C > 3$ – Stat. Indet.

where,

- R = No. of reactions
- C = No. of hinges
- 3 equations of equilibrium

Whether the structure is stable or not, must be assessed independently of its determinacy.

In general, for trusses,

$R + B < 2N$ – Unstable

$R + B = 2N$ – Stat. Det.

$R + B > 2N$ – Stat. Indet.

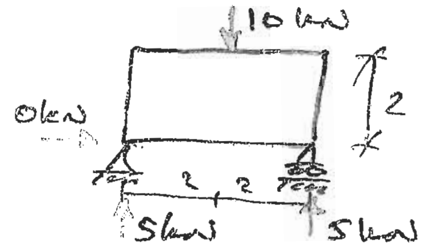
where

R – no. of reactions

B – no. of members

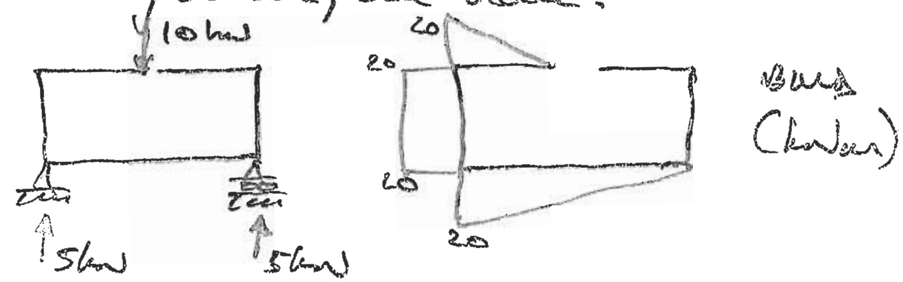
N – no. of Nodes / Joints

Structures may be internally or externally determinate or indeterminate. The following structure is externally determinate:



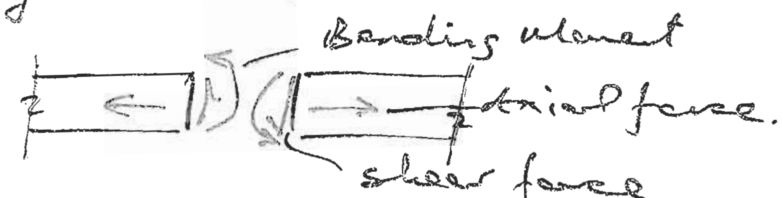
$R - C = 3$ now
⇒ Stat. Det ?
No! Externally Det.
Internally Indet.

If we "cut" this frame at the top, or anywhere, we have:

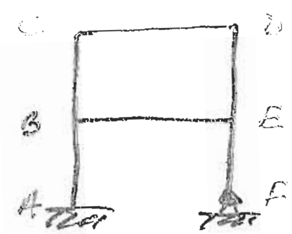


By cutting this frame, we have removed its ability to take bending moment, shear force & axial forces at the cut. We have been able to analyse the frame with this cut. Thus the original frame had 3 restraints more than the equations of statics – thus the original frame was "internally indeterminate" & its "degree of indeterminacy" is 3 (as we needed to remove 3 restraints to make it determinate)

In the above example we "cut back" the structure to determine its degree of indeterminacy; ie we removed internal redundancies or restraints over what is needed for stat. det. structures. The redundancies that can be removed in any structure are:



Example 1

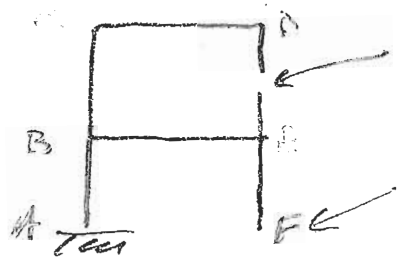


Is the structure stable, stat. det or indet? If indet. give the degree of indet.

- Externally: $R = 3^{A} + 2^{F} = 5$.

Thus it is $5-3 = 2°$ indet externally.

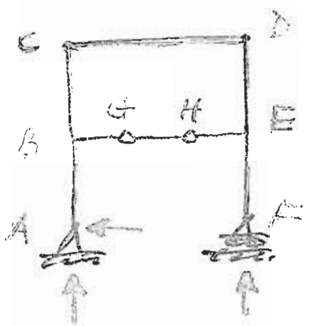
- Internally: We know from the previous example that a box is indet., thus we need to cut box BCDE once. We are still left with ABEF as a solid structure. If we remove the support @ F we won't have this problem. Thus:



The above structure is made of cantilevers & resembles a tree!

The structure is 5° indet.

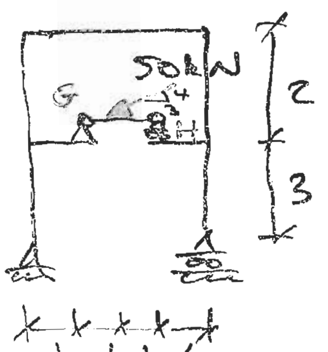
Example 2



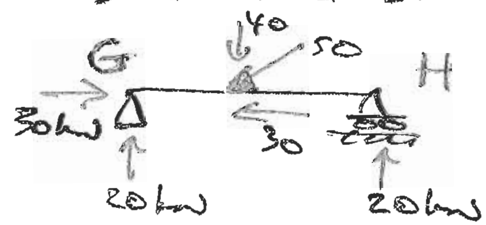
This frame is externally determinate but internally indeterminate to 1°.

To show this, remove the hinge @ H & replace with a roller (i.e. release the axial load/force transfer mechanism :

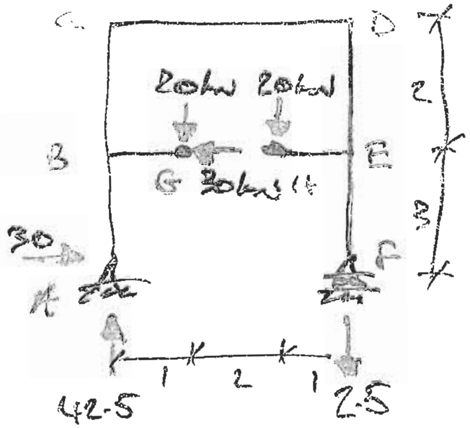
So, we have :



This frame is det. To show this we'll take the forces shown & solve.



We separate out the beam GH and analyse it. Then, we applied its reactions as actions to the rest of the frame...



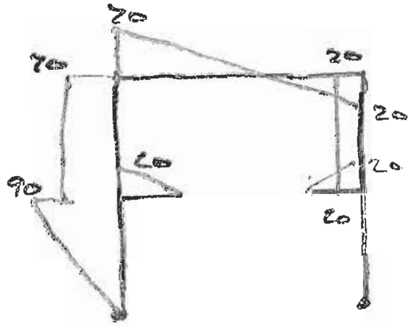
ΣM about $A = 0$

$\therefore (20 \times 1) + (20 \times 3) - (30 \times 3) + (4 \times V_F) = 0$

$\Rightarrow V_F = +2.5$ kN i.e. ↓

$\Sigma F_y = 0$

$\therefore V_A = 20 + 20 + 2.5 = 42.5$ kN



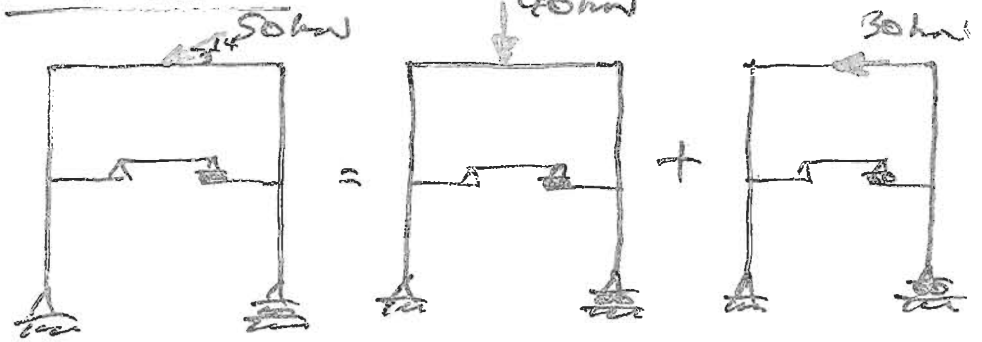
BMD (kNm)

Verify this as an exercise

We can see the effect of removing the redundancy @ H – we've been able to solve the structure.

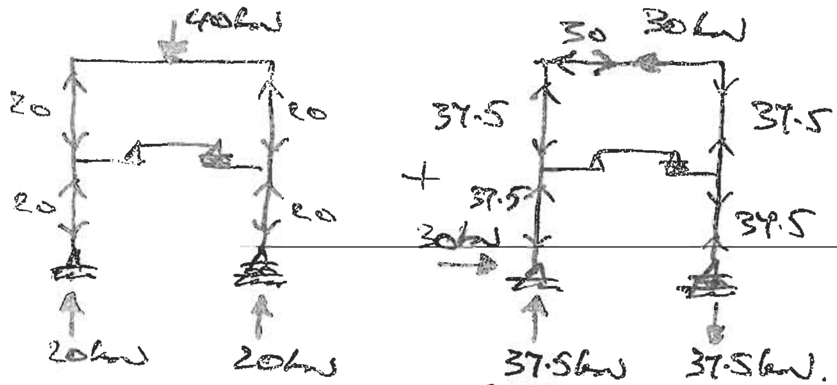
The importance of the redundancies (i.e. whether they are there or not) and how much they can affect a structure can be shown using the same frame :

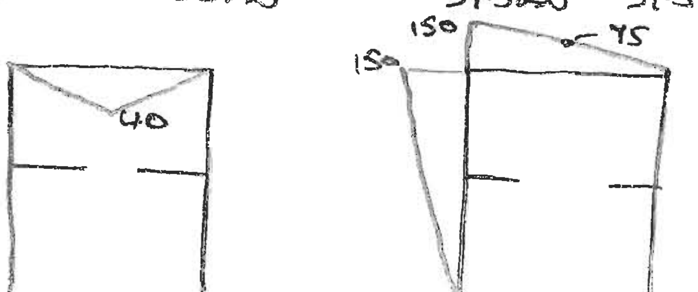
Example 3



This procedure of adding structures to get a compound structure is called <u>superposition</u> & works for all linearly elastic & linear geometric structures.

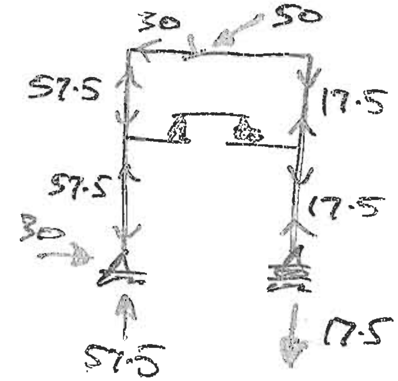
We have:

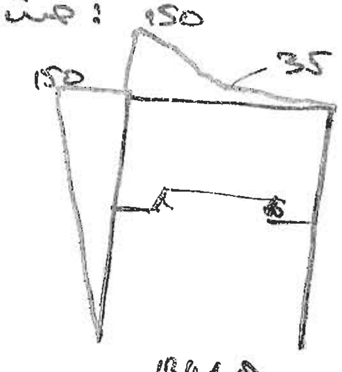




BMD (kNm) BMD (kNm)

Combine to give:





BMD (kNm)

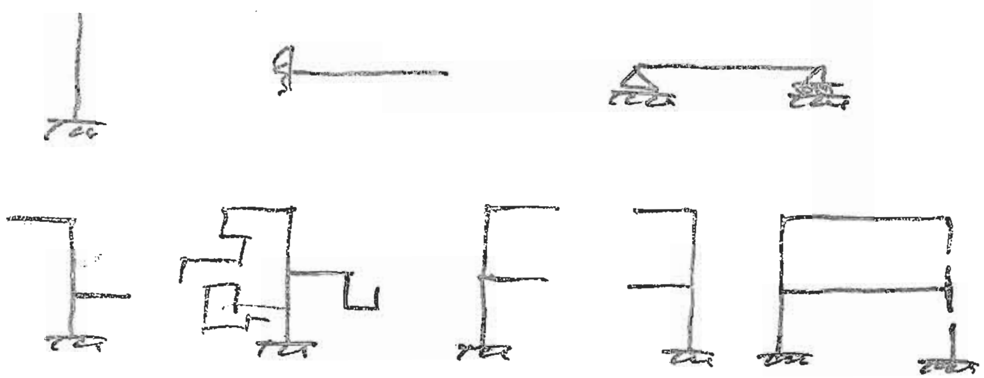
No bending moment exists in the beam GH as no load is applied to it. The other load, i.e. the applied load does not cause bending moment in the beam as no vertical force can be transmitted through G or H. If a force (V) were transmitted through H say, a moment would exist @ the pin G, which cannot be. Likewise, no horizontal force can be transmitted from G to H because of the roller at H. Also, horizontal force from E to H cannot be as they cannot pass the roller @ H.

→ Results from "cut back" of structure

→ In original & cut back structure.

Cutting Back

When we are cutting a structure back to determine its determinacy we are looking to create "base structures" which we know the behaviour/determinacy of:



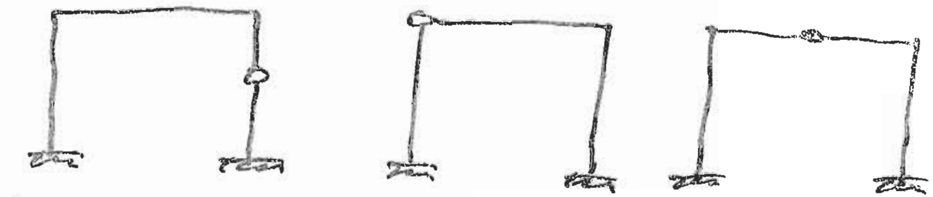
Hinges

When dealing with hinges in a structure, it is often better to re-introduce the redundancy of bending & then cut back to our base structures:

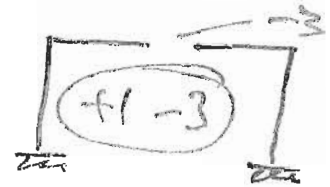
2 restraints ⇒ 5 restraints

Example 4

Determine the degree of indet.:



Add 1 restraint: 

Remove 3 restraints: 

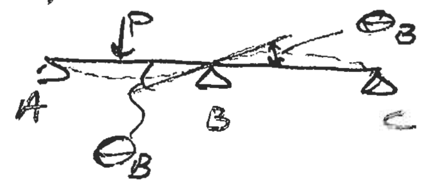
Thus all the frames shown are

$+1-3=-2 \Rightarrow 2^{\circ}$ Indet.

Analysis of Indeterminate Structures

Many different methods exist for the analysis of indeterminate structures. One branch of which, the stiffness or displacement method, relies on the compatibility conditions of geometry to aid the solution:

for example, take:



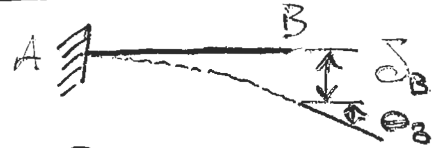
We can see ~~that~~ the rotation of the beam at B is the same for span AB or span BC. Hence if we had expressions for the rotation at B for each of the spans (in terms of load & geometry) they could be equated & used to solve ~~the~~ structure. Thus knowledge of the possible independent joint displacements is important, i.e. kinematic indeterminacy.

Kinematic Indeterminacy

Definition: The no. of <u>independent</u> joint displacements possible.

<u>Note</u> that static indet is not related to kinematic indet.

<u>Example 1:</u>



$$\delta_{xA} = \delta_{yA} = \theta_A = 0$$

$\delta_{xB} = 0$ – i.e. ignore axial deformation

$$\delta_{yB} \neq 0$$

$$\theta_B \neq 0$$

⇒ Of all possible independent joint displacements, only 2 are not known

⇒ Cantilever is 2° Kin. Indet.

i.e. 2 unknown displacements.

Also note that ~~the~~ cantilever is statically determinate.

Further, we only consider vertical movement & rotations – we ignore axial deformations.

Example 2: Fixed-fixed beam:

$\delta_{xA} = \delta_{yA} = \theta_A = 0$
$\delta_{xB} = \delta_{yB} = \theta_B = 0$

$\Rightarrow$ beam is kin. determinate.
Note it is 3° stat indet.

Example 3: Simply-supported beam:

$\delta_{xA} = \delta_{yA} = 0 \quad \theta_A \neq 0$
$\delta_{xB} = \delta_{yB} = 0 \quad \theta_B \neq 0$

$\hookrightarrow$ Neglecting axial deformation

$\Rightarrow$ Beam is kin. indet to 2°
Note it is statically determinate.

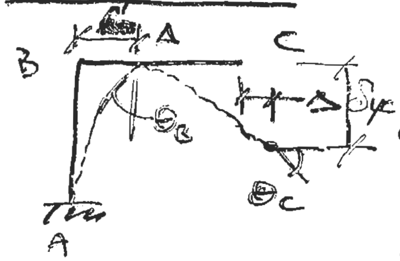
Example 4: Continuous Beam:

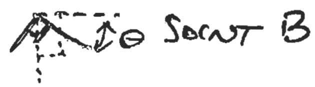
2° Stat. Indet.

$\delta_{xA} = \delta_{yA} = \delta_{xB} = \delta_{yB} = \delta_{xC} = \delta_{yC} = 0$
$\theta_A \neq 0 \quad \theta_B \neq 0 \quad \theta_C = 0$

$\Rightarrow$ 2° Kin Indet. — Neglect Axial δ

Example 5: Simple frame:





$\delta_{xA} = \delta_{yA} = \theta_A = 0$
$\delta_{xB} = \Delta$ say
$\delta_{yB} = 0$ — N.A.D.
$\theta_B \neq 0$
$\delta_{xC} = \Delta$ — NAD.
$\delta_{yC} \neq 0 \quad \theta_C \neq 0$

$\Rightarrow$ 4° Kinematically Indet.

Note that it would have been 5° but for the fact that $\delta_{xB} = \delta_{xC} = \Delta$ because we are neglecting axial deformation. As these joints are thus both related & equal to Δ they are not independent of each other & thus the only displacement that counts is Δ, by definition of kinematic indeterminacy.

i.e.

1. θ_B
2. δ_{yC}
3. θ_C
4. $\delta_{xB} = \delta_{xC} = \Delta$

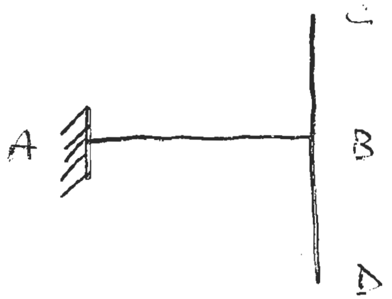
} No. of independent joint displacements

In example 5, we had 5 unknown displacements. These 5 unknowns, before analysis for relationships, are called the "degrees of freedom" of the structure. Once we had identified the relationship $\delta_{xB} = \delta_{xC} = \Delta$ we determined the kin. indet.

i.e. $$\text{D.O.F.} - \#\text{relationships} = {}^{\circ}\text{K.I.}$$

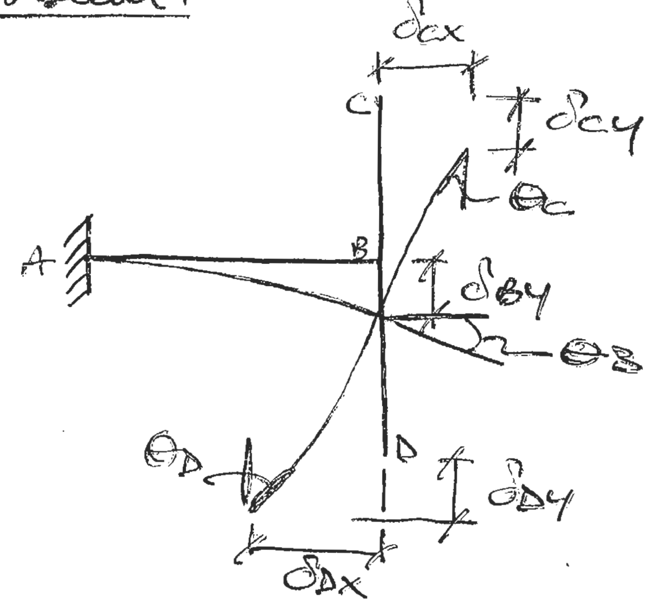
Thus, about frame 5, we would say it had 5 degrees of freedom, 1 inter-relationship & 4° of kin. indet.

Problem 1:



- State the D.O.F.
- State the °K.I.
- State your axial δ assumptions.

Problem 1



Joint	δx	δy	θ
A	0	0	0
B	0	✓Δ	✓
C	✓	✓Δ	✓
D	✓	✓Δ	✓

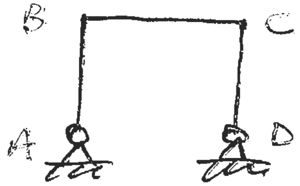
Displacement is zero, we know it is zero.

Possible disp. unknown value.

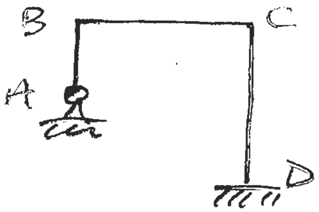
We have 8° of freedom but, if we neglect axial deformation, $\delta_{cy} = \delta_{BY} = \delta_{DY} = \Delta$, Thus we have: $\Delta, \delta_{cx}, \theta_c, \theta_B, \theta_D, \delta_{DX}$

i.e. 6° kin. indet.

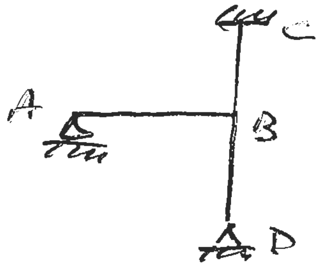
Problem 2



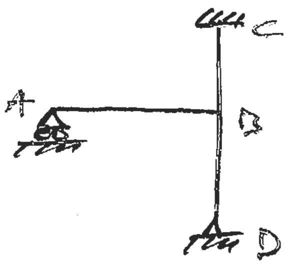
Problem 3



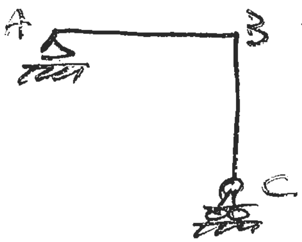
Problem 4



Problem 5



Problem 6

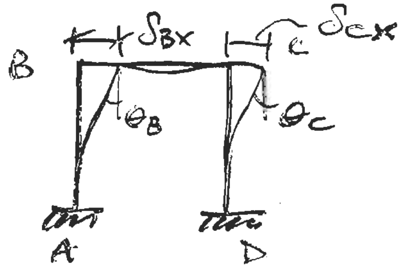


For all problems, state the DOF, show the relationships & state the °K.I.

Normally, we take the displacements that may be caused by any loading possible on the structure & use this to help us determine the °K.I.

However, in some cases, we can take the type of loading into account to aid our solution of the structure:

Example 6:



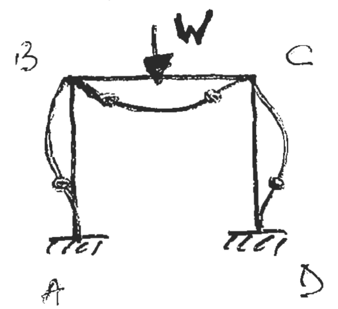
If we ignore axial deformation,
$\delta_{BY} = \delta_{CY} = 0$
Thus we have, as possible disps.:

δ_{Bx}, δ_{Cx}, θ_B, θ_C, i.e. DOF = 4

But, again ignoring axial disps,
$\delta_{Bx} = \delta_{Cx}$ – 1 relationship
$\Rightarrow$ °KI = 4 − 1 = 3° kin indet.
i.e. Δ, θ_B, θ_C

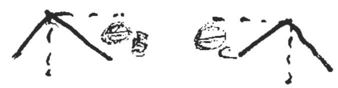
Taking the same frame & taking loading into account:

Example 7 Particular Case:



In the case of a symmetric structure w/ symmetric loads we have no sway i.e. $\delta_{Bx} = \delta_{Cx} = 0$

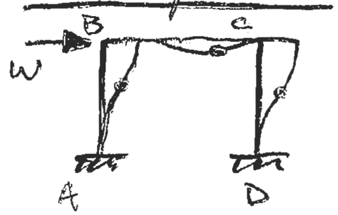
Also, as the beam BC is symmetric we have $\theta_B = -\theta_C$



Thus of our original, general structure $\Delta = 0$ & $\theta_B = \theta_C = \theta$ leaving us with a single unknown disp. θ

⇒ Kinematically indet to 1°.

Example 8 Particular Case:



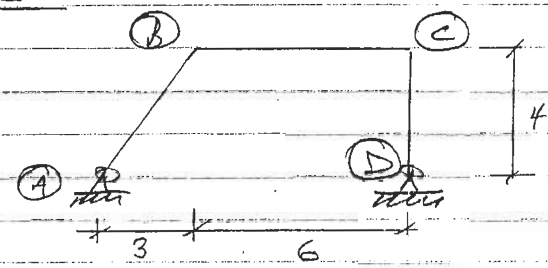
As the structure is symmetric the load will be shared equally between the two halves

⇒ The disps will be symmetric

i.e. $\delta_{Bx} = \delta_{Cx} = \Delta$ & $\theta_B = \theta_C = \theta$

⇒ Kinematically indet to 2°

EXAMPLE:



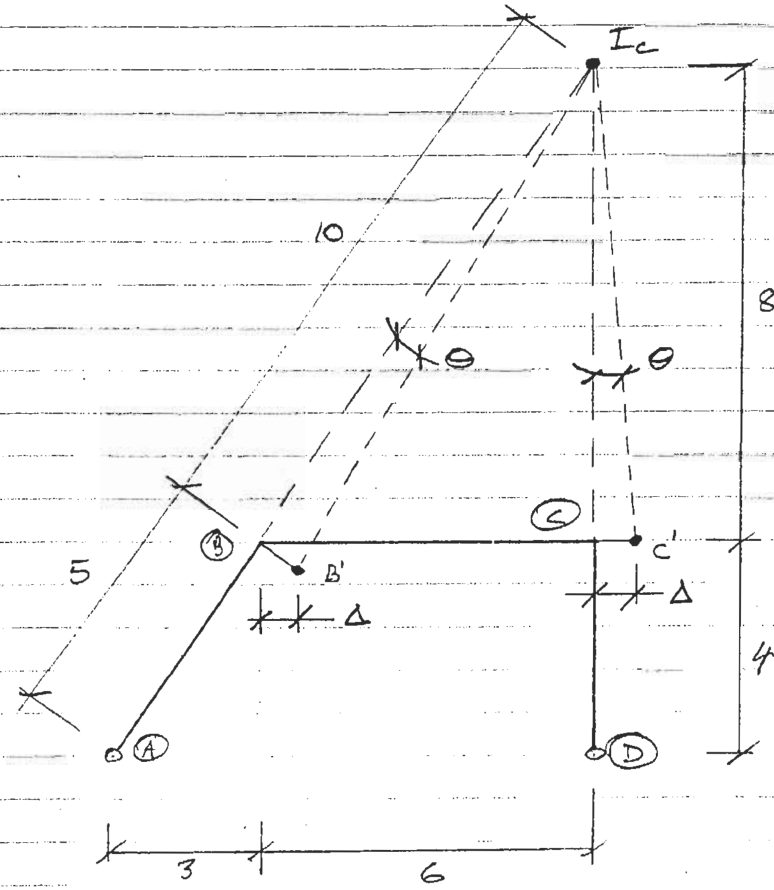
• DETERMINE THE KINEMATIC INDET.

IF MEMBERS AB & CD ARE PROJECTED UPWARDS THEY WOULD MEET AT A POINT CALLED I_c: I_c = INSTANTANEOUS CENTRE OF ROTATION.

ANY DISPLACEMENTS OF THE STRUCTURE WILL THUS TEND TO BE ROTATIONS ABOUT POINT I_c.

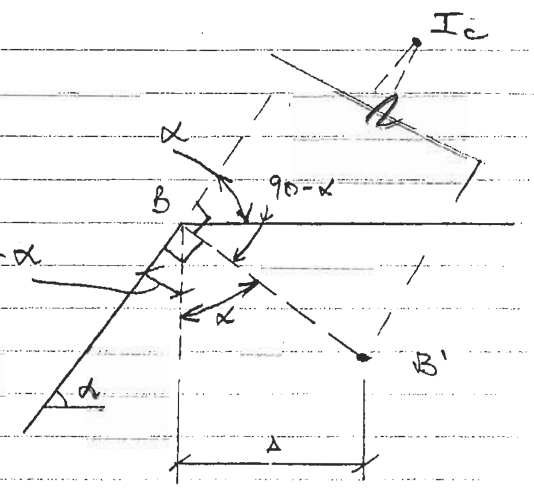
AS WE ARE NEGLECTING AXIAL DEFORMATIONS THE DISTANCE FROM B TO C i.e. MEMBER LENGTH OF BC WILL NOT ALTER THUS ANY HORIZONTAL DEFLECTION OF B WILL BE MATCHED AT C. HOWEVER, MEMBER AB CAN ONLY ROTATE ABOUT A, AS WE ARE NEGLECTING AXIAL DEFORMATIONS, THUS B WILL ALSO MOVE DOWNWARDS.

THIS IS SUMMED UP IN THE FOLLOWING DIAGRAM:



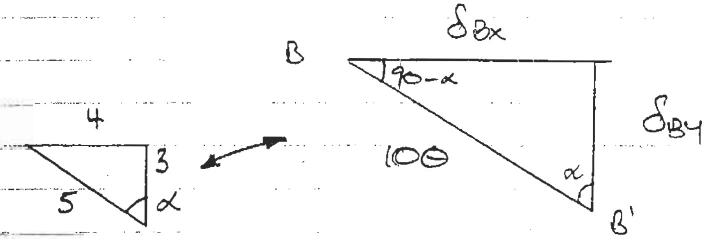
THE COLUMN I_C BC HAS ROTATED θ. THIS GIVES US TWO DEFLECTED POSITIONS FOR JOINTS B & C, B' & C' RESPECTIVELY.

FOR SMALL DISPLACEMENTS, THE LENGTH OF THE CURVE & THE CHORD ACROSS IT ARE VERY CLOSE TO EQUAL.



$|CC'| = 8\theta = \Delta \Rightarrow \Delta = 8\theta$

$|BB'| = 10\theta$



$\Rightarrow \quad \sin\alpha = 4/5 = \dfrac{\delta_{Bx}}{10\theta}$

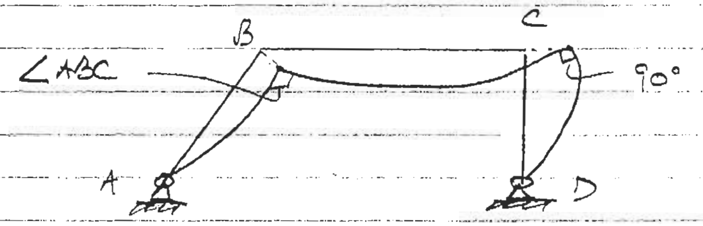
$\Rightarrow \delta_{Bx} = 8\theta = \Delta$ (AS ASSUMED)

ALSO, $\cos\alpha = \dfrac{3}{5} = \dfrac{\delta_{By}}{10\theta}$

$\Rightarrow \delta_{By} = 6\theta = \dfrac{6}{8}\Delta = 0.75\Delta$

THUS WE CAN RELATE 3 UNKNOWNS δ_{Bx}, δ_{By}, δ_{Cx} TO ONE ROTATION, θ, OR ONE DISPLACEMENT Δ ($= \delta_{Cx}$)

THE FULL DEFLECTED SHAPE IS:



SO, THE TOTAL NO. OF POSSIBLE DISPLACEMENTS ARE:

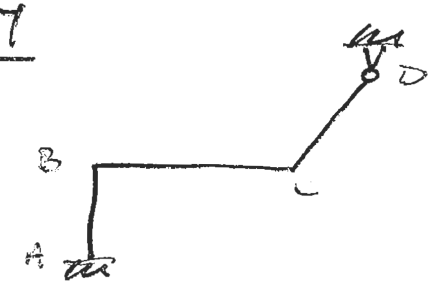
JOINT	δ_x	δ_y	θ
A	×	×	✓
B	✓	✓	✓
C	✓	×	✓
D	×	×	✓

=> 7 DEGREES OF FREEDOM.

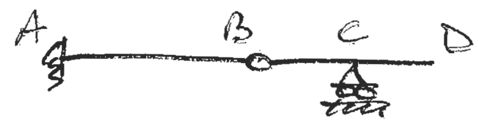
BUT AS WE HAVE SHOWN, δ_{Bx} & δ_{By} ARE RELATED TO δ_{Cx} THROUGH Δ. THUS THE NUMBER OF INDEPENDANT DISPLACEMENTS IS REDUCED BY 2

=> KINEMATIC INDET = 7 − 2 = 5°

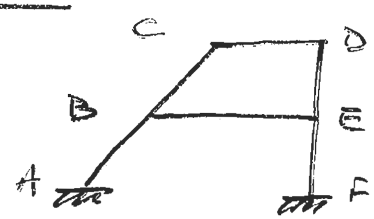
Problem 7



Problem 8



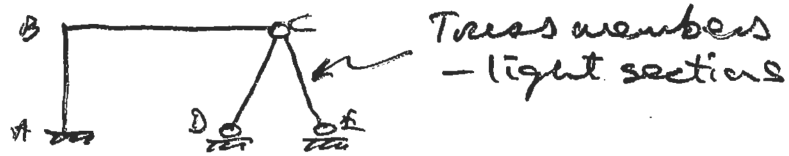
Problem 9



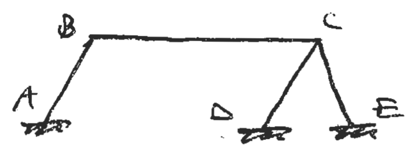
Problem 10

a)

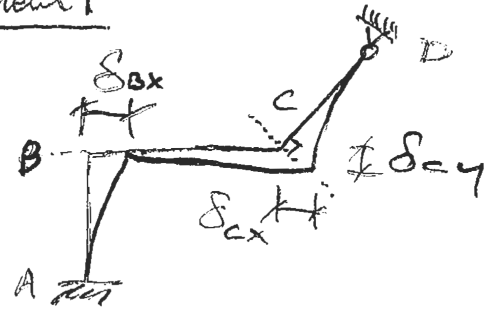
b)



c)



Problem 7

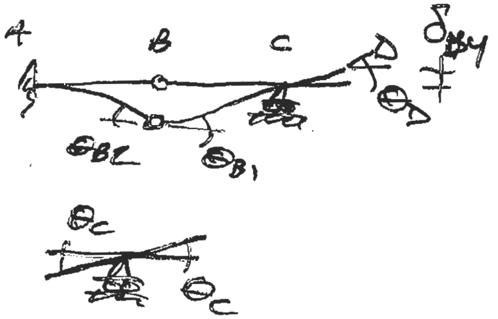


	δ_x	δ_y	θ
A	0	0	0
B	✓	0	✓
C	✓	✓	✓
D	0	0	✓

We have 6 DoF for the structure, but $\delta_{Bx} = \delta_{Cx} = \Delta$ & $\delta_{Cy} \propto \Delta$ through geometry & neglecting axial deformation.

⇒ Kinematically indet to 4°

Problem 8



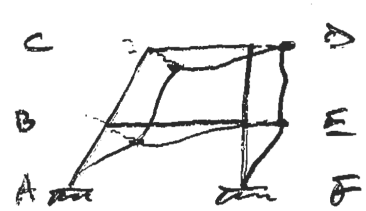
	δ_x	δ_y	θ
A	0	0	0
B	0	✓	θ_{B1} θ_{B2}
C	0	0	✓
D	0	✓	✓

We have 6DoF & no relationships

⇒ 6° K.I.

Note we essentially treat AB & BCD as two seperate structures & also when dealing with beams we have no x-displacements.

Problem 9.



	δ_x	δ_y	θ
A	0	0	0
B	✓	✓	✓
C	✓	✓	✓
D	✓	0	✓
E	✓	0	✓
F	0	0	0

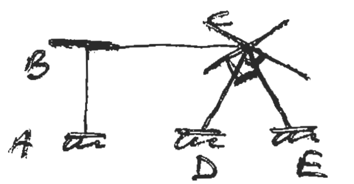
We have 10 D.O.F.

This structure will rotate about its I_c (Instantaneous centre of rotation) hence all translations can be related to the rotation @ I_c, θ_{Ic}. Joint rotations are still independent of I_c

⇒ All δ_x & δ_y's $\propto \theta_{IC}$

⇒ 5° K.I.

Problem 10 (a)

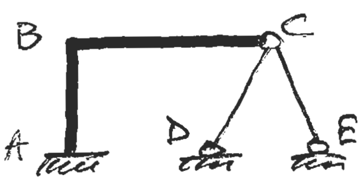


Joint C can only exist along both of the blue lines shown (i.e. potential disps of members CD & CE).

This means that C cannot move from its current location, hence by N.A.D. of BC B cannot move either. Thus we have no frame sway & only joint rotations

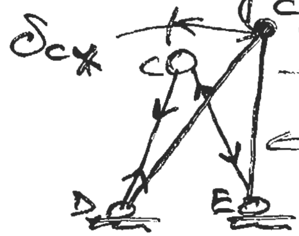
⇒ 2° K.I. θ_B & θ_C

Problem 10(b)



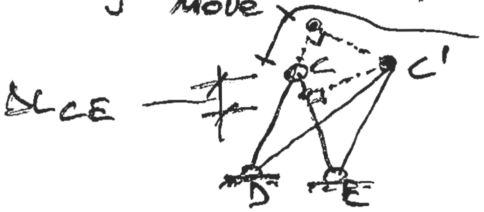
The truss members CD & CE are designed for axial load only and thus deform axially only

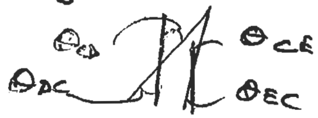
∴ The assumption of neglecting axial deformation cannot apply to these members. Take joint C:



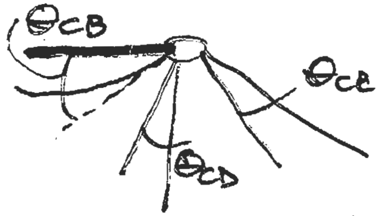
say CD longer & CE shorter

The displacements shown can occur if we consider that the joint (C) can only ~~rotate~~ move @ 90° to CD & CE;





Also, $\delta_{BX} = \delta_{CX}$ - N.A.D. of BC; $= \Delta$



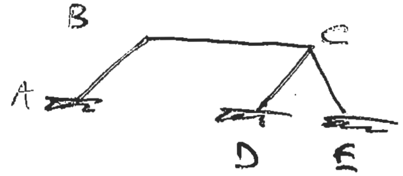
JOINT C

In total, we have

- $\Delta = \delta_{BX} = \delta_{CX}$
- δ_{CY}
- θ_{CB}
- $\theta_{CD} = \theta_{DC}$
- $\theta_{CE} = \theta_{EC}$
- θ_B

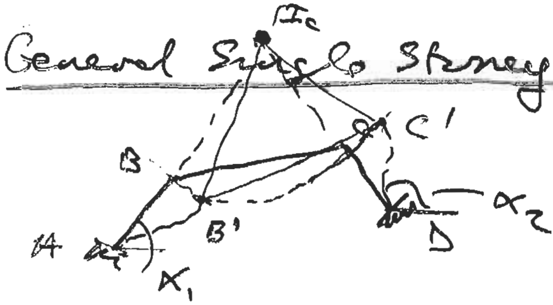
$\Rightarrow$ 6° K.I. 9 D.O.F.

Problem 10(c)

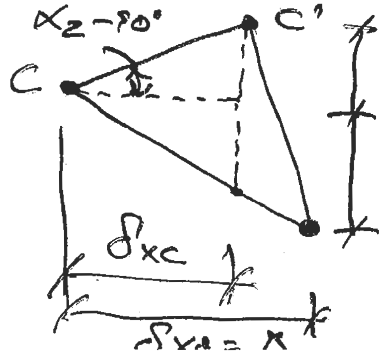


Similarly to Problem 10(a) joint C cannot translate hence we can only have rotations at the joints $\Rightarrow$ 2° K.I.

General Single Storey Frame:



1) Because BB' is ⊥ to AB $\delta_{xB} \propto \delta_{yB}$
2) Since B' is defined in space, C' is also, as BC & CD cannot change length
3) Thus $\delta_{xC} \propto \delta_{yC} \propto \delta_{xB} = \Delta$ say
4) We also have two rotations, θ_B & θ_C



As can be seen, all the displacements map onto a single triangle hence as the angles are known, all the displacements are related to Δ.

$\Rightarrow$ 3° K.I.

This is an alternative way to think about the problem.
Consideration of the θ_{IC} of the lamina $I_C BC$ leads to the same result: θ_{IC}, θ_B & θ_C
This is probably the easier method.